

## The Galilean Transformation

The Consequence of research work of Galileo on the motion of the Projectile led to formulate Galilean Transformations.

These are used to relate the motion which are observed by two observers in two different inertial frames -

Some of his results are as follows

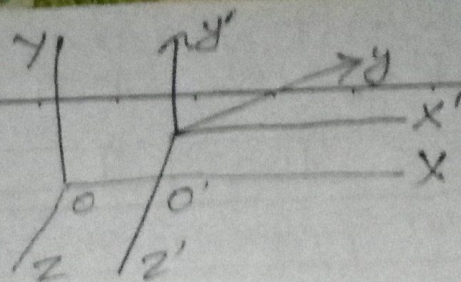
- 1) The motion of a particle projected at any angle may be derived from the motion of the particle thrown vertically upward.
- 2) If a particle is thrown straight up from a cart which is moving with uniform speed the observer on the cart may see the particle moving up & down but the motion observed by an observer on the ground may be described by superimposing the motion of the cart into that of projectile.

Consider two frames of reference one of rest and other moving with constant speed with respect to that at rest.

Suppose that there are two observers observing the event at any point P from the two frames of reference simultaneously.

Let the two frames of reference be parallel to each other that is  $x'$  axis is parallel to  $x$ -axis,  $y'$  axis parallel to  $y$ -axis and  $z'$  axis parallel to  $z$ -axis.



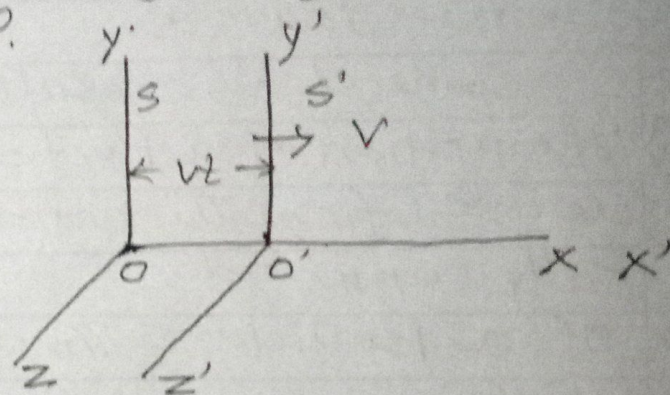


Case 1 When the 2nd frames ~~S and S'~~ is moving relative to first along +ve x-axis.

Consider two frames S and S' of references one at rest and the other is moving with uniform velocity  $v$ .

Let  $O$  and  $O'$  be the observers situated at the origins of S and S' respectively - They are observing the same event at any point P.

Let the coordinates of P be  $(x, y, z, t)$  &  $(x', y', z', t')$  relative to origins  $O$  &  $O'$  respectively.



The origin of the two frames of reference are so chosen they coincide at  $t = t' = 0$

Let the frame S' have the velocity  $v$  only in  $x'$ -direction.

Then  $O'$  has velocity  $v$  only along  $x'$ -axis.

The two systems can be combined to each other by the following equations

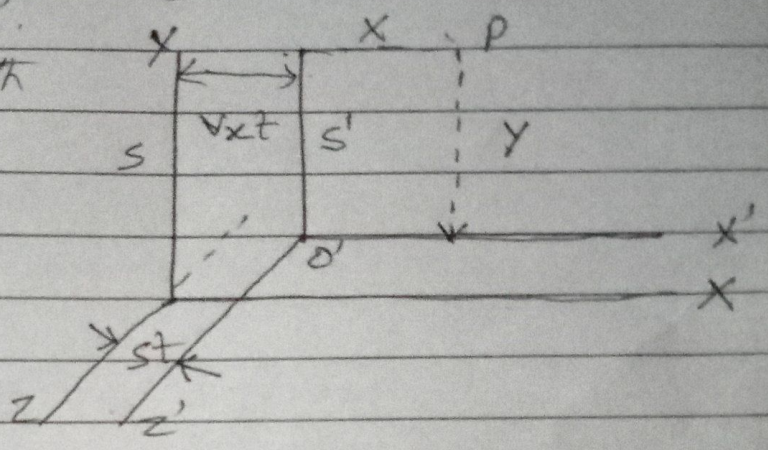
$$\left. \begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned} \right\} \text{--- (i)}$$

These are the Galilean transformation equations relating the observations of position and time made by two observers in two different inertial frames.



When the 2nd frame is moving along a straight line relative to 1st along any direction.

Consider two frames  $S$  &  $S'$ .  
The latter ( $S'$ ) moving with velocity  $V$  relative to the former ( $S$ )



Such that  
 $\vec{V} = i V_x + j V_y + k V_z$

where  
 $V_x, V_y$  &  $V_z$  are the components of  $\vec{V}$  along  $x, y$  and  $z$  axis respectively.

Let  $O$  &  $O'$  be the observers situated at the origins of  $S$  and  $S'$  respectively observing the same event at Point  $P$ .

Let the coordinates of  $P$  relate to  $O$  and  $O'$  be  $(x, y, z, t)$  and  $(x', y', z', t')$  respectively.

The origins and axes of the two frames are so chosen that they coincide at  $t = t' = 0$ .

From fig. The  $z$ -axis is perpendicular to the plane of the paper. Then after a time  $t$ , the frame  $S'$  is separated from frame  $S$  by a distance  $V_x t, V_y t$  and  $V_z t$  along  $x, y$  &  $z$  axes respectively. Then the two systems can be related by the following equations

$$\left. \begin{aligned} x' &= x - V_x t & \text{--- (i)} \\ y' &= y - V_y t & \text{--- (ii)} \\ z' &= z - V_z t & \text{--- (iii)} \\ t' &= t & \text{--- (iv)} \end{aligned} \right\} \text{--- (2)}$$